COMPRESSION OF LIMBS BY TIGHT BANDAGES: A THEORETICAL MODEL

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Abstract
Human body parts are sometimes compressed by tight clothes, footwear, bandages etc. Therefore, it is important to balance between functionality and comfort of garments in order to avoid negative effects on human health. Mechanical properties of both garment and tissue need to be taken into account for the analysis of pressure of tight garments. The thick walled tube theory is adjusted to a composite cylinder with a rigid core, as an approximate limb model with corresponding edge conditions. The results suggest the effect of tightness and elastic properties of the tissue and the tight garment on the level and type of tissue stresses.

Key words
tight clothes and footwear, tissue stress, thick walled tube

Introduction

Compressive activity on the human body occurs due to tight garments and footwear or in case of load bearing elements of equipment or devices such as e.g. pulley or similar. The application of compressive garments or elements is broad, including the f eld of medical therapy (i.e. compressive bandages), sportswear and equipment or in aesthetic garments [1, 2]. In the design of compressive bandages, it is important to understand the body part geometry along with bandage and tissue deformation characteristics [3]. Some studies concerning medical therapy applications have reported on a target pressure achievement through multi-layered bandages [4, 5]. Another study has analysed the effect of curved body part surface on compressive elastic garments pressure and the possibilities to provide the required equal pressure distribution [6].

This work is an overview of basic mechanical tools, which can be used in the analysis of body parts exposed to compressive tight clothing, belts, bandages etc. The motivation for this work is an attempt to prevent common deformations of the lower leg caused by a tight elastic reinforced socks cuff, Figure 1. This article is a continuation of a previous detailed study of the aforementioned deformation [7] by Šomođi et al.

Figure 1. Lower leg deformation by a tight sock cuf

A similar ef ect occurs due to a tightly f t belt as a piece of clothing or footwear. If we aim to analyse the transitional phase of deformation from the moment of stretching to the f nal state of equilibrium deformation, the viscoelastic (attenuating) behaviour of the tissue should be taken into consideration. The latter will not be considered here, thus the f nal equilibrium condition will be considered important. Therefore, this work will be limited to a simple linear elastic model of the material's deformational behaviour.

Cylinder compressed by tight elastic tube

We shall f rst consider a homogeneous elastic cylinder with a tightly f t thin walled elastic tube. Figure 2 gives the cross section of cylinder 1 with a tightly f t tube 2. In the initial non-deformed state, the thin tube radius is smaller than the cylinder radius for the given initial overlap, ΔR, which can be def ned as tightness.

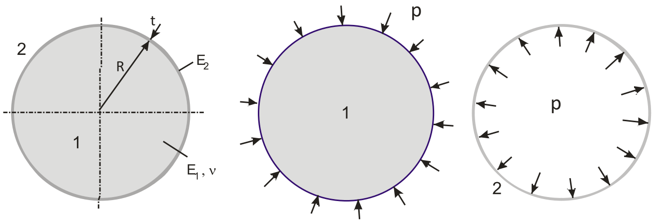

Figure 2. A set of cylinder and a tightly f t thin walled tube

In the cylinder and tube assembly, there is a mutual pressure (p) action due to overlap. The amount of this pressure is a result of the reduced radius of the cylinder and the increased radius of the tube that will compensate the initial overlap ΔR. The stress in the cylinder corresponds to the uniform double pressure ! ∀! #p, while there is a circular stress σ_φ=pR/t in the tube, as shown in Figure 3.

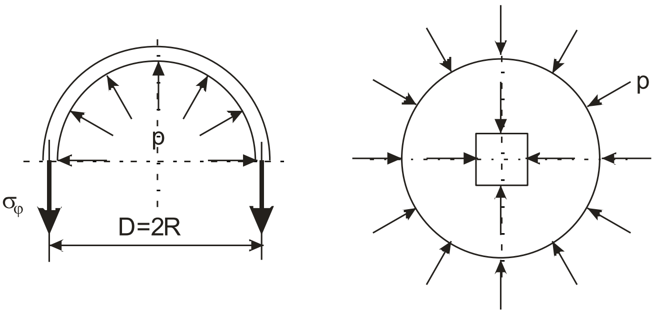

Figure 3. Stress state in a tube and a cylinder

Based on Hook's law, the cylinder and tube deformaHons are as follows:

$$\Delta R_1 = \frac{pR}{E_1}(1-\nu)\ ,\ \Delta R_2 = \frac{pR^2}{tE_2} \qquad (1)$$

Based on the deformaHon condiHons, the cylinder and tube pressure are as follows:

$$\Delta R_1 + \Delta R_2 = \Delta R \;\; => \;\; p = \frac{\Delta R}{\frac{R}{E_1}(1-\nu) - \frac{R^2}{tE_2}} \qquad (2)$$

In case of an inhomogeneous i.e. composite cylinder, or a cylinder with a rigid or hollow core, the situation becomes more complex as the stress in the cross section is not constant. In case of rotational symmetry (i.e. a set of concentric tubes), the analysis can be conducted on the basis of the theory of thick walled tubes.

Fundamental theory of thick walled tubes

The theory of thick walled tubes is a standard content of advanced mechanics of materials [8]. Thanks to the rotational symmetry in both geometry (marked area by two concentric circles) and loading (uniform internal or external surface pressure), it is convenient to use polar coordinates (Figure 4), while the stresses and displacements depend only on the radial coordinate r.

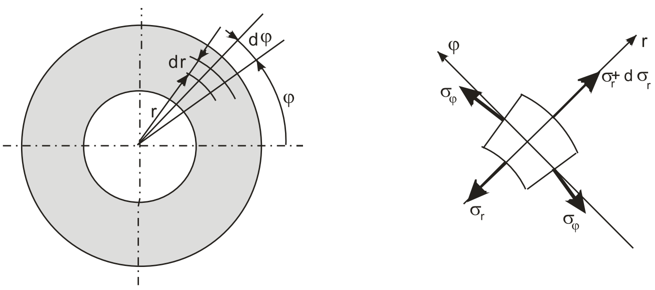

Figure 4. Thick walled tube cross section in polar coordinates and the stresses in the tube element

The static condition results from the forces equilibrium on the differential element (Figure 4). The equation of the forces equilibrium in the r direction is as follows:

$$-\sigma_r \cdot r d\varphi + (\sigma_r + d\sigma_r)(r+dr)d\varphi - \sigma_\varphi \cdot dr \sin\frac{d\varphi}{2} \cdot 2 = 0 \qquad (2)$$

The sine of a differential small angle is almost equal to the real angle. After neglecting the small value of higher order, this equation becomes as follows:

$$(\sigma_r - \sigma_\varphi)dr + r d\sigma_r = 0 \qquad (4)$$

The geometric considerations of the deformed state give the relative deformations expressions in the radial and circumferential direction depending on the radial displacement u(r):

$$\varepsilon_r = \frac{du}{dr}\ ,\ \varepsilon_\varphi = \frac{u}{r} \qquad (5)$$

Hook's law gives the relation between the stress and strain in linear elastic deformation:

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\varphi)\ ,\ \varepsilon_\varphi = \frac{1}{E}(\sigma_\varphi - \nu\sigma_r) \qquad (6)$$

When the system of equations (4, 5, 6) is solved by the unknown stress components, the result is as follows (details are not given here):

$$\sigma_r = C_1 + \frac{C_2}{r^2}\ ,\ \sigma_\varphi = C_1 - \frac{C_2}{r^2} \qquad (7)$$

The constants C_1 i C_2 come from the internal and external tube edge condition, where the radial stress corresponds to the given pressure.

As a typical example, we can consider a thick tube with an internal pressure p, whose inner and outer radius is R_1=R, R_2=2R. The f rst equation (7) gives the conditions of the inner and outer edges:

$$-p = C_1 + \frac{C_2}{R^2}\ ,\ 0 = C_1 + \frac{C_2}{4R^2} \qquad (8)$$

The solutions of the system (8) are as follows:

$$C_1 = \frac{1}{3}p\ ,\ C_2 = -\frac{4}{3}pR^2 \qquad (9)$$

The final expressions for the radial and circular stresses are:

$$\sigma_r = \frac{p}{3}\left(1 - 4\frac{R^2}{r^2}\right)\ ,\ \sigma_\varphi = \frac{p}{3}\left(1 + 4\frac{R^2}{r^2}\right) \qquad (10)$$

These expressions represent the distribution of the radial and hoop stress component on the tube cross section. The distribution is given in Figure 5. The hoop stress is tensile (positive), while the radial stress is negative, i.e. compressive. The highest value of a particular stress component is the hoop stress on the inner edge of the tube cross section,

which is σ_{max}=5/3 p.

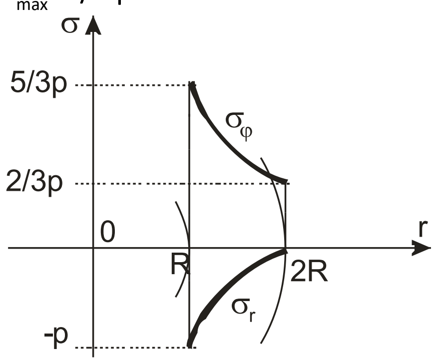

Figure 5. Distribution of the hoop and radial stresses of the internal pressure loaded tube

Technical applications are mainly based on the given theory, i.e. gun barrel loaded by an internal release pressure, and a tighten shaft-hub coupling. Here, the possibility of the thick walled tube theory is used in case of an elastic cylinder with a rigid core, which can be considered as an approximate model for the limbs loaded by an external pressure as a result of a tightly f tted clothing, footwear or similar.

Cylinder with rigid core compressed by tight tube

Due to simplicity, the limb will be modelled as a composite cylinder comprising a rigid core (bone) and an elastic external shell which represents the muscle, tendon, fat and skin tissues. The external pressure due to tightly f tted thin tube is also present.

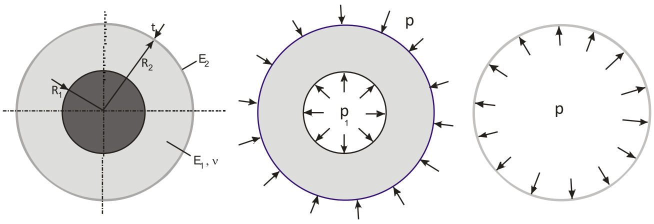

Figure 6. Assembly of a cylinder with a rigid core and a tight thin tube (left), pressures on the elastic part of the cylinder and the tube.

In the assembly, there is the pressure between the cylinder and the thin tube and pressure p_1 (Figure 6) on the inner edge of the cylinder due to core resistance.

Although the pressures p and p1 are yet unknown, the edge conditions of the thick tube – the soft tissue are as follows:

$$\sigma_r(R_2) = -p \quad , \quad \sigma_r(R_1) = -p_1 \qquad (9)$$

The deformation conditions should be met as well: on the internal edge, the radial displacement is equal to zero due to rigid core resistance, while at the external edge as earlier, the radial displacement and extension of the tightly f tted tube together correspond to the initial overlap ΔR:

$$u(R_1) = 0 \quad , \quad -u(R_2) + \Delta R_{cilovr\,2} = \Delta R \qquad (10)$$

Based on the equations (5, 6, 7), radial displacement u(r) is given as:

$$u = \frac{r}{E_1}\left[C_1(1-\nu) - \frac{C_2(1+\nu)}{r^2}\right] \qquad (11)$$

Now, finally from the equations (9, 10, 11), the expression of the pressure is as follows:

$$p = \frac{\Delta R}{\frac{R_2^2}{E_2 t} + \frac{R_2}{E_1}\left[\frac{1-\nu}{1+\left(\frac{R_1}{R_2}\right)^2\frac{1-\nu}{1+\nu}} - \frac{1+\nu}{\left(\frac{R_2}{R_1}\right)^2\frac{1+\nu}{1-\nu}+1}\right]} \qquad (12)$$

We shall conclude this chapter with an illustrative example. The Young's module E_2 and radius R_2 are given with other dependent parameters, expressed as follows: E_1=0,05E_2, R_1=0,5R_2, t=0,02R_2, ΔR=0,2R_2. We shall consider three cases with three different possible values of the Poisson's ratio: a) ν=0, b) ν=0,3, c) ν=0,5. The pressure p values calculated according to equation (12) are: a) p=0.0032258 E_2 ; b) p=0.0033753 E_2 ; c) p=0.0035135 E_2. In this case, the displacement calculation shows that approximately 80…88% of the initial overlap covers the deformation of tube 2, while the rest is due to tissue deformation. Finally, the radial and hoop stress components of the tissue cross sections are determined, as shown in the diagram of Figure 7.

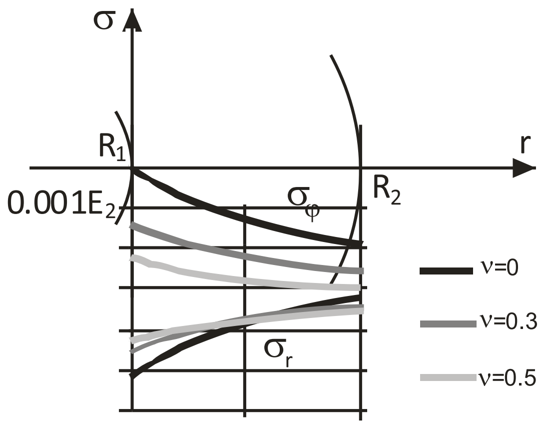

Figure 7. Stresses distribution in the results of the numerical example

The stresses are compressive. It can be observed that the values of radial stresses are greater than the hoop stresses. This difference is more pronounced in the tissue close to the bone, while the stress on the surface is closer to the uniform biaxial pressure. With the increase of the Poisson's ratio, the differences between the hoop and radial stresses in the elastic part of the cylinder reduce, as well as by the increase of the radial coordinate r. It is interesting that at ν=0, the hoop stress near the bone disappears, which happens due to the fact that the radial displacement equals to zero at this point.

Conclusion

The stresses in the tissue caused by tightly f tted clothing, footwear or similar can be approximately analysed by the application of the aforementioned mechanical models. The example of the cylinder with the rigid core requires edge conditions, which unlike the standard usage of the thick walled tube theory, uses displacements and not only stresses. In the illustrative example, the typical tissue stress distributions are given as well as the effect of the Poisson's ratio. For the purpose of a more realistic description of the situation where the pressure is due to a tightly f tted textile garment (e.g. socks), a suitable approximation of the nonlinear tensile behaviour of the textile material can be taken instead of the linear elastic behaviour of the thin tube, causing the model to become slightly complex. The model of the homogeneous soft tissue can be replaced by the more advanced model which takes into account different deformational properties of the e.g. muscle tissue and the fat under the skin tissue. In this regard, if the geometric distribution of certain types of tissue retain the axial symmetrical character, a two concentric homogeneous tube assembly of different properties E i ν can be used instead of a homogeneous tube as a soft tissue model. Finally, if the known tissue distribution with different properties along the limbs cross sections cannot be expressed as an axially symmetrical model, the experiment will require a more complex numerical modelling, i.e. a f nite element analysis.

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